Exam. Code : 103204
Subject Code : 1139

B.A./B.Sc. 4th Semester

MATHEMATICS

Paper—I

(Statics and Vector Calculus)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Do any **FIVE** questions, selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Prove that the resultant of two forces acting at a point O along OA and OB and equal in magnitude to λ OA and μ OB, respectively, is equivalent to $(\lambda + \mu)$ OC, where C is a point in AB such that $\lambda.CA = \mu.CB$.

   (b) Like parallel forces P and Q act at points L and M of a rigid body. Their resultant meets [LM] in N. When the forces are interchanged, their resultant meets [LM] in G. If LN = NG, show that P = 2Q.

2. (a) Forces equal to P, 2P and 4P act along the sides of an equilateral triangle taken in order. Find their resultant.

(b) Six coplanar forces act on rigid body along the sides AB, BC, CD, DE, EF and FA of a regular hexagon of side 1 unit. Their magnitudes are 10, 20, 30, 40, P and Q units, respectively. Find P and Q so that the system reduces to a couple and show that the moment of the couple is $75\sqrt{3}$ units.

3. (a) A body is placed on a rough plane inclined to the horizon at an angle greater than the angle of friction and is supported by a force acting at an angle θ with the inclined plane. Find the limits between which the force must lie. Also, find the least force required to support the body.

(b) A uniform ladder rests at an angle of 45° with the horizontal with its upper extremity against a rough wall and its lower extremity on the rough ground with coefficient of friction μ' and μ, respectively. Show that the least horizontal force which would move the lower extremity towards wall is $\frac{W(1+2\mu-\mu\mu')}{2(1-\mu')}$.

4. (a) D is the middle point of the base BC of a triangular lamina ABC. Show that the distance between the C.G. of ΔABD and ΔACD is $\frac{BC}{3}$.

(b) Find C.G. of a solid hemisphere.

5. (a) A square uniform plate is suspended at one of its vertices and a weight equal to half that of the plate is suspended from the adjacent vertex of the square. Show that the inclination of this side to the vertical is $\tan^{-1}\frac{1}{2}$.

(b) A straight uniform rod of weight W is suspended from a peg by two strings attached at one end to the peg and the other end to the extremities of the rod, the angle between the strings is a right angle and one is twice as long as the other, find their tensions.

SECTION—B

6. (a) Prove that the necessary and sufficient condition for the vector function $\vec{f}(t)$ to have constant magnitude is $\vec{f}\cdot\frac{d\vec{f}}{dt}=0$.

(b) Show that :

(i) $\nabla(\vec{r}\cdot\vec{a})=\vec{a}$

(ii) $\nabla[\vec{r},\vec{a},\vec{b}]=\vec{a}\times\vec{b}$, where $\vec{a}$ and $\vec{b}$ are constant vectors.

7. (a) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4). Also find maximum value of directional derivative at P(1, 2, 3).

(b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2).

8. (a) Show that the vector field represented by

$\vec{F} = (z^2 + 2x + 3y)\hat{i} + (3x + 2y + z)\hat{j} + (y + 2zx)\hat{k}$ is irrotational but not solenoidal.

(b) Verify divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} = yz\hat{k}$ taken over the curve bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

9. (a) Use Gauss divergence theorem to evaluate $\int \vec{f} \cdot d\vec{S}$, where $\vec{f} = x^3\hat{i} = y^3\hat{j} + z^3\hat{k}$ and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$.

(b) By transforming to triple integral evaluate :

$$I = \iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy),$$

where S is the closed surface bounded by the plane $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$.

10. (a) Apply Green's theorem in plane to evaluate $\oint_C [(y - \sin x)dx + \cos x dy]$, where C is the triangle enclosed by the lines $y = 0$, $2x = \pi$, $\pi y = 2x$.

(b) State and Prove Stoke's theorem.